A Layman's Guide to SUSY GUTs

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ABSTRACT

The determination of the most straightforward evidence for the existence of the Superworld requires a guide for non-experts (especially experimental physicists) for them to make their own judgement on the value of such predictions. For this purpose we review the most basic results of Super-Grand unification in a simple and clear way. We focus the attention on two specific models and their predictions. These two models represent an example of a direct comparison between a traditional unified-theory and a string-inspired approach to the solution of the many open problems of the Standard Model. We emphasize that viable models must satisfy all available experimental constraints and be as simple as theoretically possible. The two well defined supergravity models, SU(5) and $SU(5)\times U(1)$, can be described in terms of only a few parameters (five and three respectively) instead of the more than twenty needed in the MSSM model, i.e., the Minimal Supersymmetric extension of the Standard Model. A case of special interest is the strict no-scale $SU(5) \times U(1)$ supergravity where all predictions depend on only one parameter (plus the top-quark mass). A general consequence of these analyses is that supersymmetric particles can be at the verge of discovery, lurking around the corner at present and near future facilities. This review should help anyone distinguish between well motivated predictions and predictions based on arbitrary choices of parameters in undefined models.

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Contents

1	Introduction	1	
2	Two supergravity models: why these choices?	1	
3	Convergence of gauge couplings: geometry versus physics	3	
4	Constraints from unification		
5	The origin of M_{SUSY} and the need for local supersymmetry	8	
6	Details on the chosen supergravity models: SU(5) and SU(5)xU(1) 6.1 The minimal SU(5) supergravity model	13 13 14	
7	Experimental Predictions	16	
8	Conclusions	21	
Re	deferences 2		

1 Introduction

The purpose of this review paper is to present a simple guide for non-specialists into the complex world of Supersymmetry. A world with many appealing features: the boson-fermion equivalence, the unification of all gauge couplings and masses, consequently of all forces, including gravity, and the ultimate goal of discovering the Theory of Everything (TOE). The large number of papers published on this subject does not allow the possibility of everyone carefully distinguishing unjustified claims from real achievements, especially for those cases of experimental interest. It is precisely in this field that clarity is badly needed. To predict the energy level where the Superworld should show up is one of the most exciting problems of modern physics. The effort needed to implement a project, in terms of people and financial resources, is so vast that experimentalists themselves need to judge the value of a "prediction". For example, if a paper is published where the energy level is shown to be unaccessible with present facilities, those physicists engaged for years in related experiments should be able to understand and judge the real value of such predictions. The primary purpose of the present paper is to achieve this goal.

2 Two supergravity models: why these choices?

The LEP e^+e^- collider at CERN has been in operation since 1989 with a center-of-mass energy around the Z-boson mass ($M_Z \approx 91\,\text{GeV}$). During this time a vast amount of data has been collected on many different decay modes of the Z. Analyses of the totality of the data in the context of the Standard Model of electroweak interactions show no deviations from expectations, which have been tested at the level of one-loop and for some quantities two-loop corrections [1]. In particular, the weak mixing angle ($\sin^2 \theta_W(M_Z)$) and the QCD running coupling ($\alpha_3(M_Z)$) have been determined with unprecedented accuracy. This picture-perfect agreement between theory and experiment has had several consequences. One of these is the immediate rejection of large classes of models which predicted new physics below the electroweak scale.

The lesson from LEP is therefore how to reconcile the successes of the Standard Model with the compelling reasons for the existence of new physics beyond it. An obvious shortcoming of the Standard Model is the ad-hoc nature of the many parameters involved: the quark and lepton masses, the quark mixing angles, and the CP violating phase. However, even if explanations for these parameters could be found within physics not so far from the electroweak scale, a problem would show up when extrapolating the theory to much larger energies. This so-called gauge hierarchy problem is manifest in theories with elementary scalar fields, such as the Higgs boson in the Standard Model. The reason is that radiative corrections to the Higgs-boson mass become extremely large if the theory also contains very massive particles, as is the case in unified theories, or any theory which attempts to incorporate gravity in a common framework. This is a "problem" for the theory, as opposed to a termi-

nal disease, in that there is a grossly undesirable solution which involves fine-tuning parameters to large numbers of decimal places, order-by-order in perturbation theory.

Particle physics seems to always find solutions to its problems by invoking the existence of larger symmetries. The proposed solutions to the above two problems of the Standard Model are no different: unified models and supersymmetry, respectively. The former postulate that the Standard Model is actually embedded in a larger group structure which includes $SU(3)_C \times SU(2)_L \times U(1)_Y$ but which is only fully manifest at very large energies, where new degrees of freedom are excited and the symmetry group is effectively enlarged to e.g., SU(5), $SU(5) \times U(1)$, SO(10). In the larger theory relations among the parameters arise, which help explain some of the regularities observed in the Standard Model parameters, such as the quantization of the electric charge and some relations among the quark and lepton masses. In fact, the existence of three light generations of quarks and leptons had already been predicted in this context in studies of the m_b/m_τ ratio [2]. An immediate consequence of the larger symmetry is that the gauge couplings of the Standard Model are "unified", i.e., take the same value at the unification scale (E_{GUT}) , even though they are measured to be different numbers at the weak scale. By-products of great experimental interest are the new interactions among the quarks and leptons which may be induced by the heavy degrees of freedom. Among these, the most model-independent one leads to the prediction of proton decay, which must occur at a rather slow rate $(\tau_p \gtrsim 10^{32} \,\mathrm{y})$ to avoid conflict with experimental limits.

Supersymmetry solves the gauge hierarchy problem by predicting the existence of partners for the ordinary particles which differ by half-a-unit of spin, e.g., electron($\frac{1}{2}$)-selectron(0), photon(1)-photino($\frac{1}{2}$). The radiative corrections to the Higgs-boson mass are now tamed by the fact that the partners of the heavy particles have an opposite effect on the correction. However, since supersymmetry cannot be an unbroken symmetry (otherwise the sparticles would have the same mass as their Standard Model partners and no such sparticles have been observed) the cancellation is not perfect, but up to the typical mass splitting of particles and their superpartners. If this splitting can be predicted to be no larger than $\sim 1 \text{ TeV}$, then the gauge hierarchy problem will remain solved. In fact, it is this further requirement which demands that we enlarge the symmetry even more, by making supersymmetry a local symmetry called *supergravity*. In this class of theories the mass splittings can be explicitly computed in terms of very few parameters. This ability is of great experimental interest, since a large class of (e.q., collider) and rare low-energy processes can be calculated in terms of few parameters, providing close knit correlations among the several experimental predictions, which may otherwise appear arbitrary.

We will consider two such supergravity models: the minimal SU(5) [3] and $SU(5) \times U(1)$ [4]. Conceptually, the SU(5) model is a typical (and the simplest) grand unified supersymmetric model which predicts the existence of proton decay at a rate which should be on the verge of being observed. In fact, its non-supersymmetric ancestor, the Georgi-Glashow SU(5) model [5], was ruled out experimentally on the basis of its incorrect (too short) prediction for the proton lifetime. Theoretically, this model also suffers from the gauge hierarchy problem, which is present in the

absence of supersymmetry. In the supersymmetric SU(5) model, the unification of gauge couplings occurs at a scale $E_{GUT} \sim 10^{16} \,\text{GeV}$, and the proton lifetime problem of non-supersymmetric SU(5) is naturally solved. As we now well know, unification of the gauge couplings also does not work in non-supersymmetric SU(5), while it works very well in its supersymmetric counterpart. The goodness of the prediction for the m_b/m_τ relation alluded to above is also maintained with the addition of supersymmetry [2].

The $SU(5) \times U(1)$ supergravity model is best motivated in the context of superstrings, where it is found that $SU(5) \times U(1)$ (in contrast with SU(5)) is relatively easy to obtain. In this model the unification of couplings should occur at the string scale ($E_{GUT} \sim 10^{18} \,\text{GeV}$). This fact appears to require a non-minimal spectrum of particles at intermediate scales, in contrast with the SU(5) model where the "big desert" picture is assumed to hold. Furthermore, the $SU(5) \times U(1)$ supergravity model allows a further reduction of the unknown parameters of the theory, making its predictions very sharp. Moreover, these two models allow to compare possible implementations of supersymmetry: one (SU(5)) as example of traditional unified theories, the other $(SU(5) \times U(1))$ as example of string-inspired theories.

3 Convergence of gauge couplings: geometry versus physics

The three gauge couplings of the Standard Model, as measured at the Z-boson mass scale, can be expressed in terms of: α_e , $\sin^2 \theta_W$, and α_3 . Experimentally we know that (see e.g., [6]),

$$\alpha_e^{-1}(M_Z) = 127.9 \pm 0.1,$$
 (1)

$$\alpha_3(M_Z) = 0.120 \pm 0.010,$$
 (2)

$$\sin^2 \theta_W(M_Z) = 0.2324 \pm 0.0006. \tag{3}$$

The $U(1)_Y$ and $SU(2)_L$ gauge couplings are related to these by $\alpha_1 = \frac{5}{3}(\alpha_e/\cos^2\theta_W)$ and $\alpha_2 = (\alpha_e/\sin^2\theta_W)$. In numbers:

At
$$Q = M_Z$$

$$\begin{cases} \alpha_1^{-1} = 58.91 \pm 0.16 \\ \alpha_2^{-1} = 29.72 \pm 0.080 \\ \alpha_3^{-1} = 8.33 \pm 0.69 \end{cases}$$
 (4)

The three gauge couplings evolve with increasing values of the scale Q in a logarithmic fashion, and may become equal at some higher scale, signaling the possible presence of a larger gauge group. However, this need not be the case: the three gauge couplings may meet and then depart again. Conceptually, the presence of a unified group is essential in the discussion of unification of couplings. In this case, the newly excited degrees of freedom will be such that all three couplings will evolve together for scales

 $Q > E_{GUT}$, and one can then speak of a unified coupling. (For a recent review and extensive references to the current literature see e.g., Ref. [7].)

The running of the gauge couplings is prescribed by a set of first-order non-linear differential equations: the renormalization group equations (RGEs) for the gauge couplings. In general, there is one such equation for each dynamical variable in the theory (i.e., for each gauge coupling, Yukawa coupling, and sparticle mass). These equations give the rate of change of each dynamical variable as the scale Q is varied. For the case of a gauge coupling, the rate of change is proportional to (some power of) the gauge coupling itself, and the coefficient of proportionality is called the beta function. The beta functions encode the spectrum of the theory, and how the various gauge couplings influence the running of each other (a higher-order effect). Assuming that all supersymmetric particles have a common mass M_{SUSY} , the RGEs (to two-loop order) are:

$$\frac{d\alpha_i^{-1}}{dt} = -\frac{b_i}{2\pi} - \sum_{j=1}^3 \frac{b_{ij}\alpha_j}{8\pi^2}.$$
 (5)

where $t = \ln(Q/E_{GUT})$, with Q the running scale and E_{GUT} the unification mass. The one-loop (b_i) and two-loop (b_{ij}) beta functions are given by

$$b_i = \left(\frac{33}{5}, 1, -3\right),$$
 (6)

$$b_{ij} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix}. \tag{7}$$

These equations are valid from $Q = M_{SUSY}$ up to $Q = E_{GUT}$. For $M_Z < Q < M_{SUSY}$ an analogous set of equations holds, but with beta functions which reflect the non-supersymmetric nature of the theory (i.e., with all the sparticles decoupled),

$$b_i' = \left(\frac{41}{10}, -\frac{19}{6}, -7\right),\tag{8}$$

$$b'_{ij} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}. \tag{9}$$

The non-supersymmetric equations are supplemented with the initial conditions given in Eq. (4).

If the above is all the physics which is incorporated in the study of the convergence of the gauge couplings, then it is easy to see that the couplings will always meet at some scale E_{GUT} , provided that M_{SUSY} is tuned appropriately [8, 9, 10, 11]. This is a simple consequence of euclidean geometry, as can be seen from Eq. (5). Neglecting the higher-order terms, we see that as a function of t, α_i^{-1} are just straight lines. In fact, the slope of these lines changes at $Q = M_{SUSY}$, where the beta functions change. The convergence of three straight lines with a change in slope is then guaranteed by euclidean geometry, as long as the point where the slope changes is tuned appropriately. (This fact was pointed out by A. Peterman and one of us (A. Z.) in 1979 [12].)

What is non-trivial about the convergence of the couplings is that with the initial conditions given in Eq. (4), the change in slope needs to be $M_{SUSY} \sim 1 \text{ TeV } [10]$.

This prediction [10] for the likely scale of the supersymmetric spectrum (i.e., $M_{SUSY} \sim 1 \,\mathrm{TeV}$ [10]) is in fact incorrect [13]. The reason is simple: the physics at the unification scale, which is used to predict the value of M_{SUSY} , has been ignored completely. In fact, such a geometrical picture of convergence of the gauge couplings is physically inconsistent, since for scales $Q > E_{GUT}$ the gauge couplings will depart again. One must consider a unified theory to be assured that the couplings will remain unified, as shown in Fig. 1. This entails the study of a new kind of effect, namely the influence on the running of the gauge couplings of the degrees of freedom which are excited near the unification scale (i.e., the heavy threshold effects) [8, 14]. In fact, the whole concept of a single unification point needs to be abandoned. The upshot of all this is that the theoretical uncertainties on the values of the parameters describing the heavy GUT

particles are such that the above prediction for M_{SUSY} [10] is washed out completely [13]. Furthermore, the insertion of a realistic spectrum of sparticles at low energies (as opposed to an unrealistic common M_{SUSY} mass) blurs the issue some more [8, 13]. Thus, it is perfectly possible to obtain acceptable unification, with supersymmetric particle masses as low as experimentally allowed. The most complete analysis of a unified theory is shown in Fig. 2. Note the unification of the gauge couplings which continues above E_{GUT} . Notice also that "light" and "heavy" thresholds have been duly accounted for, plus other detailed effects like the evolution of gaugino masses (EGM). This effect has in fact been calculated at two loops [15].

A related point is that LEP data do not uniquely demonstrate that the gauge couplings must unify at a scale $E_{GUT} \sim 10^{16} \,\text{GeV}$ [14]. This is probably the simplest conclusion one could draw. However, this conclusion is easily altered by for example considering models with particles at intermediate scales, *i.e.*, by populating the "big desert". In fact, one such simple modification allows the gauge couplings to converge at the string scale $E_{GUT} \sim 10^{18} \,\text{GeV}$ instead [4].

4 Constraints from unification

The convergence of the gauge couplings implies that given α_e and α_3 , one is able to compute the values of $\sin^2 \theta_W$, the unification scale E_{GUT} , and the unified coupling α_U . In lowest-order approximation (*i.e.*, neglecting all GUT thresholds, two-loop effects, and taking $M_{SUSY} = M_Z$) one obtains

$$\ln \frac{E_{GUT}}{M_Z} = \frac{\pi}{10} \left(\frac{1}{\alpha_e} - \frac{8}{3\alpha_3} \right), \tag{10}$$

$$\frac{\alpha_e}{\alpha_U} = \frac{3}{20} \left(1 + \frac{4\alpha_e}{\alpha_3} \right), \tag{11}$$

$$\sin^2 \theta_W = 0.2 + \frac{7\alpha_e}{15\alpha_3} . {12}$$

Figure 1: The convergence of the gauge couplings $(\alpha_1, \alpha_2, \alpha_3)$ at E_{GUT} is followed by the unification in a unique α_{GUT} above E_{GUT} . The RGEs include the heavy and light thresholds plus the evolution of gaugino masses. These results are obtained using as input the world-average value of $\alpha_3(M_Z)$ and comparing the predictions for $\sin^2\theta(M_Z)$ and $\alpha_{em}^{-1}(M_Z)$ with the experimental results. The χ^2 constructed using these two physically measured quantities allows to get the best E_{GUT} , $\alpha_{GUT}(E_{GUT})$, and $\alpha_3(M_Z)$ (shown). The RGEs can go down to M_Z without any need of introducing a change of slope at $E\approx 10^3$ GeV as would be required if the various effects mentioned above are neglected.

Figure 2: This is the best proof that the convergence of the gauge couplings can be obtained with M_{SUSY} at an energy level as low as M_Z . Notice that the effects of "light" and "heavy" thresholds have been accounted for, as well as the Evolution of Gaugino Masses [14,15]. This figure is Fig. 2 of ref. [16]. E_{SU} is the string unification scale.

These equations provide a rough approximation to the actual values obtained when all effects are included. Nonetheless, they embody the most important dependences on the input parameters. In Fig. 3 (from Ref. [11]) we show the relation between E_{GUT} and α_3 for various values of $\sin^2 \theta_W$. One can observe that:

$$\alpha_3 \uparrow \Rightarrow E_{GUT} \uparrow \text{ for fixed } \sin^2 \theta_W$$
 (13)

$$\alpha_3 \uparrow \Rightarrow E_{GUT} \uparrow \text{ for fixed } \sin^2 \theta_W \qquad (13)$$

$$\sin^2 \theta_W \uparrow \Rightarrow E_{GUT} \uparrow \text{ for fixed } \alpha_3 \qquad (14)$$

$$\alpha_3 \uparrow \Rightarrow \sin^2 \theta_W \downarrow \text{ for fixed } E_{GUT} \qquad (15)$$

$$\alpha_3 \uparrow \Rightarrow \sin^2 \theta_W \downarrow \text{ for fixed } E_{GUT}$$
 (15)

These are the most important systematic correlations, which are not really affected by the neglected effects. These correlations are evident in Eqs. (10–12) and in Fig. 3. In this figure we also show the lower bound on E_{GUT} which follows from the proton decay constraint. Clearly a lower bound on $\alpha_3(M_Z)$ results, which allows the world-average value. Another interesting result is the anticorrelation between E_{GUT} and M_{SUSY} . This is shown in Fig. 4, where for fixed $\sin^2 \theta_W(M_Z)$ we see that increasing $\alpha_3(M_Z)$ increases E_{GUT} (as already noted in Eq. (13)) and decreases M_{SUSY} . Taking for granted this approach (i.e., all supersymmetric particle masses degenerate at M_{SUSY}) for comparison with the large amount of papers published following this logic, in Fig. 5 we see the narrow band left open once the experimental limits on τ_p and M_{SUSY} are imposed. Figure 5 is a guide to understand the qualitative interconnection between the basic experimentally measured quantities, $\alpha_3(M_Z)$, $\sin^2\theta_W(M_Z)$, τ_p , M_{SUSY} and the theoretically desirable E_{GUT} . The experimental lower bounds on the proton lifetime $(\tau_p)_{exp}$ and on M_{SUSY} produce two opposite bounds (lower and upper, respectively) on the unification energy scale E_{GUT} . Note that, in order to make definite predictions on the lightest detectable supersymmetric particle, a detailed supergravity model is needed. The study of the correlations between the basic quantities, as exemplified in this figure, is interesting but should not be mistaken as example of prediction for the superworld. In particular, the introduction of the quantity M_{SUSY} is really misleading.

5 The origin of M_{SUSY} and the need for local supersymmetry

The calculations which we have described so far, attempted to determine the value of M_{SUSY} , or more properly the supersymmetric particle spectrum, by fitting the spectrum to obtain the "best possible" unification picture. This program did not succeed because of the large inherent uncertainties in the physics at the GUT scale. Nevertheless, for a given GUT model, it should be possible to compute the GUT threshold effects and attempt the "best fit" procedure to deduce the corresponding light supersymmetric spectrum. This picture is not very satisfying since one would like to know why the supersymmetric spectrum should be the way the fit would require it to be. In other words, the real question is: what determines the values of

Figure 3: The unification scale E_{GUT} versus $\alpha_3(M_Z)$ for various values of $\sin^2\theta_W(M_Z)$ within $\pm 2\sigma$ of the world-average value. Also indicated is the lower bound on E_{GUT} from the lower limit on the proton lifetime.

Figure 4: The unification scale E_{GUT} versus M_{SUSY} for different values of $\alpha_3(M_Z)$ and fixed $\sin^2\theta_W(M_Z)$. Note the anticorrelation between M_{SUSY} and E_{GUT} . The experimental lower bound on M_{SUSY} is shown. The lower bound on E_{GUT} from Fig. 1 is also indicated.

Figure 5: The correlation between all measured quantities, $\alpha_3(M_Z)$, $\sin^2\theta_W(M_Z)$, τ_p , the limits on the lightest detectable supersymmetric particle (here represented by M_{SUSY}) and the unification energy scale E_{GUT} .

the sparticle masses? And why should these be below $\sim 1\,\mathrm{TeV}$, so that the gauge hierarchy problem is not re-introduced?

As mentioned in Sec. 2, considering a theory with supergravity (instead of global supersymmetry) provides the means to compute the masses of the sparticles [3]. This framework assumes that supersymmetry breaking occurs in a "hidden sector" of the theory, where "gravitational particles" (those introduced when the supersymmetry was made local) may grow vacuum expectation values (vevs) which break supersymmetry spontaneously in the hidden sector. These vevs are best understood as induced dynamically by the condensation of the supersymmetric partners of the hidden sector particles when the gauge group which describes them becomes strongly interacting at some large scale. The splitting of the particles and their partners would then be generated, and would be the order of the condensation scale ($\sim 10^{12-16} \, \mathrm{GeV}$). However, such huge mass splittings will not be immediately transmitted to the "observable" (the normal) sector of the theory, since the two sectors only communicate through gravitational interactions. The dampening in the transmission mechanism is such that the splittings in the observable sector are usually much more suppressed than those in the hidden sector, and suitable choices of hidden sectors may yield realistic low-energy supersymmetric spectra. This picture of hidden and observable sectors becomes completely natural in the context of superstrings, where models typically contain both sectors and one can study explicitly the predicted spectrum of supersymmetric particles at low energies.

In a large number of models, the supersymmetric particle masses at the unification scale are also "unified". This situation is called universal soft-supersymmetry-breaking, and the masses of all scalar partners (e.g., squarks and sleptons) take the common value of m_0 , the gaugino (the partners of the gauge bosons) masses are given by $m_{1/2}$, and there is a third parameter (A) which basically parametrizes the mixing of stop-squark mass eigenstates at low energies. The breaking of the electroweak symmetry is obtained dynamically in the context of these models, through the so-called radiative electroweak symmetry breaking mechanism, which involves the top-quark mass in a fundamental way [17]. After all these well motivated theoretical ingredients have been incorporated, the models depend on only five parameters: $m_{1/2}, m_0, A$, the top-quark mass (m_t) , and the ratio of the two Higgs vacuum expectations values $(\tan \beta)$ [18].

A rather interesting framework occurs in the so-called no-scale scenario [19, 20, 17], where all the scales in the theory are obtained from just one basic scale (i.e., the unification scale or the Planck scale) through radiative corrections. These models have the unparalleled virtue of a vanishing cosmological constant at the tree-level even after supersymmetry breaking, and in their unified versions predict that the universal scalar masses and trilinear couplings vanish (i.e., $m_0 = A = 0$) and the universal gaugino mass $(m_{1/2})$ is the only seed of supersymmetry breaking. Moreover, this unique mass can be determined in principle by minimizing the vacuum energy at the electroweak scale. The generic result is $m_{1/2} \sim M_Z$ [19, 20], in agreement with theoretical prejudices (i.e., "naturalness"). Furthermore, no-scale supergravity is obtained in the infrared limit of superstrings [21].

More generally, in generic supergravity models the five-dimensional parameter space is constrained by phenomenological requirements, such as sparticle and Higgs-boson masses not in conflict with present experimental lower bounds, a sufficiently long proton lifetime, a sufficiently old Universe (a cosmological constraint on the amount of dark matter in the Universe today), various indirect constraints from well measured rare processes, etc. In addition, further theoretical constraints can be imposed which give m_0 and A as functions of $m_{1/2}$, and thus reduce the dimension of the parameter space down to just three. In what follows we will focus on some specific supergravity models which are so constrained that precise experimental predictions can be made.

6 Details on the chosen supergravity models: SU(5) and SU(5)xU(1)

The two supergravity models that we have chosen belong to the class of models we just described. However, these models are even more predictive than the run-of-the-mill supergravity model because further well motivated theoretical assumptions are made.

6.1 The minimal SU(5) supergravity model

In this model the gauge group is SU(5), which completely contains the Standard Model as a subgroup. This implies that all three Standard Model gauge couplings should evolve from low energies and become equal at (and above) the unification scale E_{GUT} (up to heavy threshold corrections). This scale has some dependence on the uncertainties on the various parameters; one usually obtains $E_{GUT} \sim 10^{16} \,\text{GeV}$. The Standard Model particles and their superpartners are assigned to the $\overline{\bf 5}$ and $\bf 10$ representations:

$$\bar{\mathbf{5}} = \{d^c, L = (\begin{array}{c} e \\ \nu_e \end{array})\}, \qquad \mathbf{10} = \{Q = (\begin{array}{c} u \\ d \end{array}), u^c, e^c\}.$$
 (16)

The heavy GUT particles which are excited near the unification scale include the **24** representation of gauge bosons (and gauginos) which contains the twelve Standard Model gauge bosons (8 gluons, W^{\pm}, Z, γ) plus twelve heavy (charged and colored) gauge bosons which mediate the SU(5) gauge interactions. There is also a **24** representation of Higgs bosons (and their superpartners), and the vacuum expectation value of the neutral component of this set effects the $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry breaking. The minimal SU(5) spectrum also includes a pair of pentaplet representations ($\mathbf{5}, \overline{\mathbf{5}}$) which contain the two Higgs doublets of the low-energy supersymmetric theory, but also a pair of colored Higgs triplets.

Perhaps the most decisive property of this model is the prediction for the proton lifetime. Proton decay can be mediated in two ways: by exchange of heavy

gauge or Higgs bosons (dimension-six operators), or by the exchange of heavy Higgsinos (dimension-five operators) [22, 23]. The dimension-six operators predict a proton lifetime proportional to M_U^4 or M_H^4 respectively, and the expected values of these mass scales make this contribution rather small. On the other hand, the dimension-five operators entail a proton lifetime proportional to $M_{\widetilde{H}}^2$, which requires $M_{\widetilde{H}} > E_{GUT}$ and some strong constraints on the supersymmetric spectrum (if it is light enough to be observable). In fact, the three-dimensional soft-supersymmetry-breaking parameter space $(m_{1/2}, m_0, A)$ is constrained in such a way that the squarks and sleptons should be heavy, while the neutralinos and charginos should be light [24].

Another important constraint on the parameter space of the minimal SU(5) supergravity model is provided by cosmological considerations. The models we consider include a discrete symmetry called R-parity which has value +1 for the ordinary particles, and -1 for the supersymmetric particles. This implies that supersymmetric particles must always be created or destroyed in pairs, i.e., at each vertex there are always none, two, or four supersymmetric particles. In particular, the lightest supersymmetric particle (LSP) must be stable in these models. Astrophysical considerations require this particle to be neutral and colorless [25]. The only two candidates are the lightest neutralino (a linear combination of the weakly interacting neutral sparticles: photino, zino, two Higgsinos) and the sneutrino. It turns out that it is the neutralino which is the lightest one. Since the neutralinos are stable, they must have pair-annihilated away quite efficiently in the early Universe, otherwise their present relic abundance would be too large, and the Hubble parameter would correspond to a Universe younger than the oldest known stars.

The cosmological constraint can be usually satisfied as long as the supersymmetric particles are not too heavy. However, the proton decay constraint requires heavy squarks and sleptons and thus the constraint is relevant. In fact, the only particles which mediate pair-annihilation efficiently are the Z-boson and the lightest Higgs boson, which are light. In practice, the neutralino mass must be near half the masses of these particles so that resonant s-channel annihilation occurs [26]. The resulting effect on the parameter space of the model is quite severe [27].

6.2 The SU(5)xU(1) supergravity model

The $SU(5) \times U(1)$ ("flipped SU(5)") gauge group [28] differs from SU(5) in several ways. The Standard Model particles are also assigned to the $\overline{\bf 5}$ and $\bf 10$ representations, but in a "flipped" way (i.e., $u^c \leftrightarrow d^c$, $e^c \leftrightarrow \nu^c$) relative to the minimal SU(5) case,

$$\bar{\mathbf{5}} = \{u^c, L = \begin{pmatrix} e \\ \nu_e \end{pmatrix}\}, \qquad \mathbf{10} = \{Q = \begin{pmatrix} u \\ d \end{pmatrix}, d^c, \nu^c\}, \qquad \mathbf{1} = e^c. \tag{17}$$

Note that a right-handed neutrino ν^c has appeared naturally. This leads to an automatic *see-saw* mechanism to generate small neutrino masses in this model [29]. The heavy GUT spectrum of fields includes the **24** representation of gauge boson plus the U(1) gauge boson of $SU(5) \times U(1)$. Unlike SU(5), the symmetry breaking Higgs fields

are contained in the $10,\overline{10}$ representations (which have neutral components because of the "flipped" assignment). This property is central to the appeal of $SU(5) \times U(1)$ as a paradigm of a string model [30], since larger Higgs representations (like the 24) are not easily obtainable in string model building. There is also a pair of Higgs pentaplets $5,\overline{5}$ which contain the two Higgs doublets of the low-energy theory, and the heavy Higgs triplets.

Proton decay is a concern that is easily dismissed in the typical $SU(5) \times U(1)$ models: the dimension-six proton decay operators are small as usual, while the perilous dimension-five operators are strongly suppressed. The latter is a direct consequence of the $SU(5) \times U(1)$ symmetry as it applies to the solution of the doublet-triplet splitting problem. The Higgs triplets are not only heavy but also cannot mediate the dangerous proton decay diagram [29, 31].

The model we have studied is most simply understood in the context of string model building when the gauge couplings of the Standard Model unify at the string scale $E_{GUT} \sim 10^{18} \,\mathrm{GeV}$ [32]. This entails the addition of another 10, $\overline{10}$ representations to the heavy GUT spectrum, otherwise the gauge couplings would unify as in the minimal SU(5) model. (Specific string models with this property have also been constructed [33].)

For the soft-supersymmetry-breaking parameters we have used two stringinspired scenarios, which allow one to determine two of the parameters as functions of the third one. We have considered,

- (i) the no-scale model: $m_0 = A = 0$
- (ii) the dilaton model: $m_0 = \frac{1}{\sqrt{3}} m_{1/2}$, $A = -m_{1/2}$ [34].

Note that the dimension of the parameter space is now reduced to just three: m_t , $\tan \beta$, and $m_{1/2}$, where $m_{1/2} \propto m_{\tilde{g}}$. This implies that the sparticle masses are (up to $\tan \beta$ -dependent effects) proportional to the gluino mass. The corresponding approximate proportionality coefficients are shown in Table 1. For both these supersymmetry

Table 1: The approximate proportionality coefficients to the gluino mass, for the various sparticle masses in the two supersymmetry breaking scenarios considered for $SU(5) \times U(1)$ supergravity.

	no-scale	dilaton
$ ilde{e}_R, ilde{\mu}_R$	0.18	0.33
$ ilde{ u}$	0.18 - 0.30	0.33 - 0.41
$2\chi_1^0, \chi_2^0, \chi_1^{\pm}$	0.28	0.28
$ ilde{e}_L, ilde{\mu}_L$	0.30	0.41
\widetilde{q}	0.97	1.01
$ ilde{g}$	1.00	1.00

breaking scenarios there is a yet more restrictive possibility which we call the *strict* no-scale and the special dilaton scenarios. A further requirement allows one to deduce the value of $\tan \beta$ as a function of m_t and $m_{1/2}$, yielding highly predictive two-parameter models.

With these choices for the supersymmetry breaking parameters, one determines that the cosmological constraint does not restrict the models in any way [32, 35]. In fact, these two models offer a natural explanation for the required amount of dark matter in the Universe.

7 Experimental Predictions

As remarked above, in a generic supergravity model one expects a large degree of correlation among the predictions for, e.g., the sparticle and Higgs masses and the rates for the various experimental processes of interest. These generic models are further constrained by the non-observation of any sparticle or Higgs boson. Interestingly enough, such five-parameter models are still not very predictive in that many possible trends of correlations are possible. In contrast, in the two specific supergravity models described in Sec. 6, the correlations become sharp and of much more experimental interest.

Without imposing any further constraints on the models, besides the present collider bounds on sparticle and Higgs masses and the proton decay and cosmological constraints as applicable, we have calculated rates for the following processes at the indicated facilities:

All the above processes are kinematically accessible in the $SU(5) \times U(1)$ models. In the minimal SU(5) model the sleptons are heavy and neither the slepton pair production at LEPII nor any of the indicated processes at HERA are allowed.

We now give a sample of the actual results obtained for the most important processes listed above. These are shown in Figures 6,7,8,9.

More recently it has been realized that indirect experimental constraints on the $SU(5) \times U(1)$ models exist and can be quite significant. These constraints come

Figure 6: The number of trilepton events at the Tevatron per $100\,\mathrm{pb}^{-1}$ in the minimal SU(5) model and the no-scale $SU(5)\times U(1)$ model (for $m_t=130\,\mathrm{GeV}$). Note that with $200\,\mathrm{pb}^{-1}$ and 60% detection efficiency it should be possible to probe basically all of the parameter space of the minimal SU(5) model, and probe chargino masses as high as $175\,\mathrm{GeV}$ in the no-scale model. Upper bounds on the trilepton cross section $(\sigma \cdot B < (0.6-1)\,\mathrm{pb})$ have been recently announced by the CDF [40] and D0 [41] Collaborations.

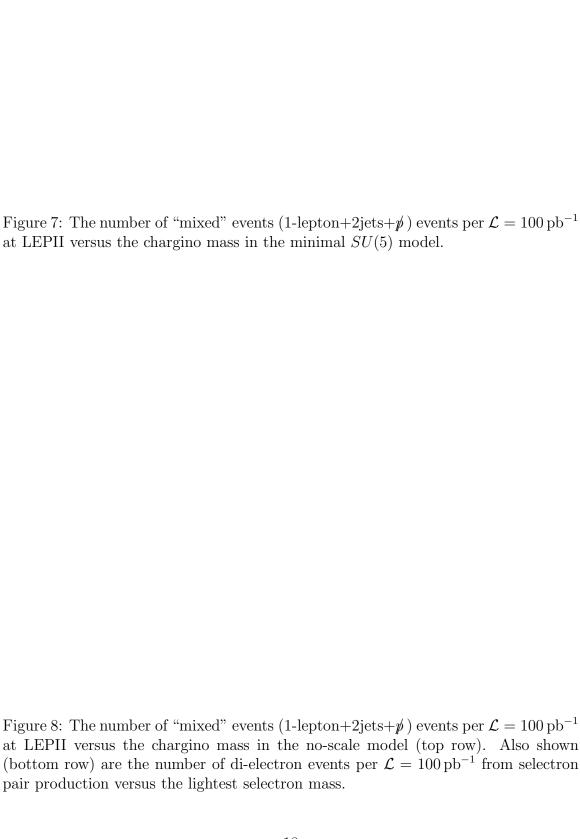


Figure 9: The elastic plus deep-inelastic total supersymmetric cross section at HERA $(ep \to \text{susy} \to eX + \not p)$ in the no-scale model versus the lightest selectron mass $(m_{\tilde{e}_R})$ and the sneutrino mass $(m_{\tilde{\nu}})$. The short- and long-term limits of sensitivity are expected to be 10^{-2} pb and 10^{-3} pb respectively.

generally from three sources: (i) the experimentally allowed range for $B(b \to s\gamma)$ as recently determined by the CLEO Collaboration [42]; (ii) the long-standing value for the anomalous magnetic moment of the muon [43]; and (iii) the precision LEP measurements of the electroweak parameters in the form of the allowed range for the ϵ_1 parameter [44]. (Only the last constraint is relevant in the minimal SU(5) supergravity model because of its relatively heavier spectrum.)

The first two constraints exclude regions of the parameter space of the $SU(5) \times U(1)$ models which span all allowed values of the chargino mass, when viewing the parameter space in the $(m_{\chi_1^{\pm}}, \tan \beta)$ plane for fixed m_t . On the other hand, the ϵ_1 constraint basically implies an upper bound on m_t : $m_t \lesssim 165\,\text{GeV}$, unless the chargino is very light $(m_{\chi_1^{\pm}} \lesssim 70\,\text{GeV})$ in which case the upper bound on m_t can be relaxed up to $m_t \lesssim 180\,\text{GeV}$. As an example of the effect of the constraints, in Fig. 10 we show the parameter space for $m_t = 130\,\text{GeV}$ (when the ϵ_1 constraint is not restrictive) for the no-scale model [45]. Clearly the region of parameter space

Figure 10: The parameter space of the no-scale $SU(5)\times U(1)$ supergravity model for $m_t=130\,\mathrm{GeV}$. Points denoted by periods satisfy all presently known experimental constraints, whereas those denoted by pluses violate the limits on $B(b\to s\gamma)$ and those denoted by crosses violate the limits on $(g-2)^{susy}_{\mu}$.

accessible to LEPII searches $(m_{\chi_1^\pm} \lesssim 100\,{\rm GeV})$ has become quite constrained.

8 Conclusions

In sum, we have presented a simplified tour of supersymmetric unified theories which hopefully will allow non-experts in the field to get acquainted with such a topical subject. One of our goals was to show that supersymmetric particles can well be "around the corner" and at the verge of discovery at present and near future facilities, such as the Tevatron, HERA, and LEP (I and II). We have also shown that there are many experimental constraints on supersymmetric unified models which need to be consistently imposed to speak about experimentally viable models. Finally, these experimentally testable models should be as simple as the best theoretical motivations allow. There is little to be learned from generic models whose many parameters can be tuned to predict anything. In fact, predicting anything is akin to predicting nothing.

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